# THE HISTORY OF THE COMPUTER

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## Simple Mechanical Calculators

Although the modern computer owes relatively little to its mechanical predecessors, we shall interpret the history of the computer as being the history of mechanical devices for performing, automatically, useful sequences of arithmetic operations. This takes us back to the first simple adding machine, which, as far as we know at present, is that of Wilhelm Schickard, 1623. Before saying something about Schickard's machine, it is interesting to consider whether the date of its invention, the early seventeenth century, is to be considered as being early, late, or in some sense appropriate.

The technological basis of a simple adding machine is the gear-wheel. It is now known that quite complicated gearing existed before the beginning of our era. The principal evidence for this comes from some heavily corroded lumps of bronze recovered from the floor of the Mediterranean in 1901 along with other objects from the remains of a Greek ship which sank near the island of Antikythera in about the year 70 B.C. Working from X-ray photographs of these pieces of bronze, Prof. de Solla Price has recently elucidated the nature of the mechanism to which they belonged. He finds that this was a geared calendrical computer, which even included an epicyclic gear, probably used for generating the cycle of the lunar months (Fig. 1). Professor Price has suggested that the geared Islamic astrolabes which were eventually transmitted to mediaeval Europe may have been an inheritance from this much earlier Greek tradition of astronomical mechanism.

The weight-driven church clock appeared in Europe soon after 1300, and the spring-driven domestic clock during the fifteenth century. Most of these early clocks were relatively crude, but there were some noteworthy exceptions; in particular, the compact astronomical clock of Giovanni de Dondi of Padua, completed in 1364. This remarkable clock had seven dials for indicating the motions of the moon and the planets, as well as showing -- incidentally -- the time of day. De Dondi has left us a description, with detailed drawings, of the complicated gearing which was used for the various planetary motions. A portion of a modern reconstruction of de Dondi's clock is shown in Fig. 2.

Although there seems to have been some degeneration in the art of clock-making for a period following the peak represented by the de Dondi clock, it seems probable that a skilled clock-maker could, if asked, have constructed a simple model of an adding machine well before the seventeenth century. Why was this not done? One possibility, of course, is that it was in fact done, and that some long-forgotten record of the event will eventually come to light, just as Schickard's description of his calculator was brought to light only three hundred years after his machine was built. However, it is also possible that the idea of mechanizing the "mental" activity of arithmetic calculation was one which could not arise until the general world-view had itself become more mechanistic.

Wilhelm Schickard (1592-1635) was professor of Biblical Languages, and later also of Astronomy, in the University of Tübingen. He was a mathematician and an expert cartographer. He knew Kepler well, and followed Kepler in being one of the first people to make use of the logarithms invented by Napier in 1614. In 1623 Schickard wrote to Kepler to describe a calculating machine which he had built, saying that he was enclosing a sketch of the machine. However, it was not until 1957 that the letter to Kepler and the sketch were reunited. A search of Kepler's papers then led to the discovery of another sketch by Schickard and some notes addressed by him to the mechanic, Johan Pfister, who had built the machine. These two sketches are shown in Fig. 3. From these documents it is possible to make a plausible reconstruction of the device (Fig. 4). Schickard regarded his machine as a four-operation arithmetic calculator; but in fact only the operations of addition and subtraction were truly mechanical.

The upper part of the machine contained six vertical cylinders each carrying a copy of the multiplication table up to 9 times 9. On pulling aside one of the nine horizontal slides, the product of a given multiplicand by one of the digits 1 to 9 would be exposed in a row of apertures. Below these cylinders was a projecting section of the machine containing the adding mechanism; and below this was a horizontal shelf containing a set of passive memory discs with digit apertures, on which the successive digits of a quotient could be recorded one by one as they were obtained during division.

The adding mechanism in Schickard's machine is likely to have been as shown in Fig. 5. The "carry" from one stage to the next would be performed by the wheels having only one tooth, which would have been placed alternately in front of, and behind, the other wheels. The intermediate gears provide a needed reversal of direction of rotation. Numbers to be added would be set up, digit by digit, by inserting a stylus into one of the ten holes in each front disc and then pulling it round until it reached a fixed stop, as in a telephone dial. The gearing was reversible and could therefore be used for subtraction as well as addition.

Although the tone of Schickard's letter to Kepler is enthusiastic, it is clear from his notes to the mechanic that the first model of the machine was badly made and unreliable in action. It is not known whether a second model was ever built, nor is there any evidence that Schickard's machine was known to his contemporaries. It is therefore unlikely that Blaise Pascal (1623-1662) had heard of Schickard when he started his extensive experiments on the construction of an adding machine during the 1640's. Pascal's first working machine probably dates from about 1645. This too was a stylus-operated machine, incorporating an ingenious carrying mechanism operated by a pivoted claw, which was lifted by the figure wheel as it approached the 9 position, and dropped when the wheel passed from 9 to 0. This mechanism was more sophisticated than that of Schickard, but not necessarily more reliable. Several of the machines built under Pascal's direction are still in existence.

Although both Schickard and Pascal proposed that their machines should be used for multiplication by repeated addition, this process is unacceptably slow on a stylus-operated machine. It was left to

Gottfried Leibniz (1646-1716) to design a calculating machine which could, in principle, be used for reasonably fast multiplication. In Leibniz's machine each turn of a handle at the front of the machine would cause the multi-digit factor set up on a sliding carriage to be added into a stationary result register. This machine worked by the "stepped wheel" principle used in many desk calculating machines up to the present day. The carrying mechanism of Leibniz's machine was not, however, completely automatic, requiring intervention by the user in certain cases.

Quite apart from their limited speed, the calculating machines of Pascal and Leibniz were not sufficiently well made to be used for real calculation. Further, since such machines could be built only by skilled craftsmen they were prohibitively expensive -- collector's pieces rather than true arithmetical aids. This was also true of their successors, the often highly ingenious calculating machines invented during the eighteenth, and even the early nineteenth century. Hand-operated calculators did not become reliable, easy to use, and relatively inexpensive, until towards the end of the nineteenth century. Early in the twentieth century electric motors were added to the more expensive models, and the four operations of addition, subtraction, multiplication and division were at last made completely automatic. However, it is probably true to say that, until cheap pocket calculators became available in about the year 1974, the greater part of the world's miscellaneous calculations in workshops, design offices or laboratories was done with pencil and paper supplemented by a slide-rule and books of mathematical tables.

## Difference Engines

We have seen that, from the middle of the seventeenth century onwards, it would have been widely known that the four fundamental operations of arithmetic could, at least in principle, be performed by machinery. We therefore turn our attention to the next stage in the evolution of the computer, namely the linking together of several arithmetic operations into a completely automatic sequence.

Mathematical tables computed with pencil and paper had always contained many errors, and it was recognized that these would not be eliminated by the use of a calculating machine as long as it was still necessary to set up by hand every number occurring during the calculation, and to write down on paper the result of every arithmetic operation. Only a completely automatic machine which could print its results would eliminate these errors. A possible basis for such a machine was found in the method of finite differences already employed by professional computers.

The "difference table" of a mathematical function consists of a series of columns of numbers. The first column contains values of the function corresponding to equally spaced values of its argument; the second contains the differences $\Delta f$ between successive pairs of numbers in the first column; the third contains the differences $\Delta^2 f$ between successive second differences, and so on. Using the notation $f_m = f(x_0 + mh)$, $m = 1, 2, 3, \ldots$, where $h$ is a constant argument increment, and defining

$$\Delta^m f_r = \Delta^{m-1} f_{r+1} - \Delta^{m-1} f_r, \quad r = 1, 2, \ldots ,$$

the first five columns of a typical difference table might start with the values shown below:

$$\begin{array}{ccccc}
f_0 & & & & \\
 & \Delta f_0 & & & \\
f_1 & & \Delta^2 f_0 & & \\
 & \Delta f_1 & & \Delta^3 f_0 & \\
f_2 & & \Delta^2 f_1 & & \Delta^4 f_0 \\
 & \Delta f_2 & & \Delta^3 f_1 & \\
f_3 & & \Delta^2 f_2 & & \Delta^4 f_1 \\
 & \Delta f_3 & & \Delta^3 f_2 & \\
f_4 & & \Delta^2 f_3 & & \\
 & \Delta f_4 & & & \\
f_5 & & & &
\end{array}$$

If the function f is a polynomial of degree n, the nth differences are constant. If the function is non-polynomial but smooth, the numbers in successive columns of differences become steadily smaller. For purposes of table-making, it is usually possible to find some column of differences whose values remain effectively constant over a fairly large range of argument values. In this case, additional sloping lines in the difference table can be built up by performing a sequence of additions from right to left. For example, if the fifth differences in the table above are assumed to be negligible, the fourth differences will be constant, and we may set $\Delta^4 f_2 = \Delta^4 f_1$. The numbers $\Delta^3 f_3$, $\Delta^2 f_4$, $\Delta f_5$, $f_6$ can then be calculated according to the following scheme:

$$\begin{array}{ccccc}
 & & & & \Delta^4 f_1 \\
 & & & \Delta^3 f_2 & \\
 & & \Delta^2 f_3 & & \Delta^4 f_2 \\
 & \Delta f_4 & & \Delta^3 f_3 & \\
f_5 & & \Delta^2 f_4 & & \\
 & \Delta f_5 & & & \\
f_6 & & & &
\end{array}$$

This process is liable to be unstable in the sense that the inevitable rounding errors are amplified in going from line to line; therefore additional "guard digits" must be carried during the calculation. Also the work has to be restarted fairly frequently using accurately computed values of the differences. In spite of these disadvantages, the method lends itself to the routine calculation of mathematical tables because the main part of the work consists of a uniformly repetitive sequence of additions (or subtractions if the sign is negative) which can be performed by mathematically unskilled labour.

The first person to suggest that the process of constructing tables from differences could be mechanized seems to have been J.H. Müller (1746-1830), a military engineer from Darmstadt. In a brief appendix to a book published in 1786 describing his newly invented circular adding machine, Müller states that he would be prepared to construct a new kind of calculating machine which would be able to compute, and also print, sequences of numbers whose initial differences were given. As far as is known, Müller never obtained the promise of financial support which was his stated condition for disclosing the details of his proposal.

## Babbage's Difference Engine

Charles Babbage (1791-1871) was an energetic innovator and reformer in the scientific life of Victorian England. In 1821 he decided that mathematical tables could be computed more easily and accurately by mechanical means (i.e. by a difference

engine) than by hand. To verify the principle, he assembled a small hand-operated non-printing model which, judging from Babbage's description, had one figure wheel for a constant second difference, two figure wheels for the first difference, and three wheels for the function value. With this machine he was able to compute the 30 values of $x^2 + x + 41$, $x = 1(1)30$, in two and a half minutes. (The values of this function for $x = 1, 2, \ldots, 39$ are all prime.)

In 1822 Babbage proposed that the British government should subsidize the building of a large twenty-digit difference engine working from constant sixth differences. The engine would automatically set up the type from which tables were to be printed. This proposal was accepted by the government in 1823, and Babbage engaged an able mechanical engineer, Joseph Clement, to supervise the construction. However, the work went ahead only very slowly. Babbage had decided that a reliable engine would have to be large and solidly built; as a result, Clement spent much of his time developing improved machine tools, an activity which significantly advanced the level of Victorian engineering, but which necessarily delayed the construction of the difference engine. Further, Babbage was continually revising, but never finalizing, his ideas on the details of the engine, especially those of the all-important "carry" mechanism whose speed determined that of the whole machine. This led him eventually to the idea of the "anticipatory carry", which was a mechanical implementation of a method nowadays used in electronic arithmetic circuits.

A six-digit demonstration model of the difference engine, working from constant second differences, was assembled in 1832 from components intended for the big machine (Fig. 7). Then in 1833 all work on the difference engine stopped, first because of dispute about the way Clement was to be paid, and then because of delays in moving the workshop to a new site adjoining Babbage's house. During this period of enforced waiting, which lasted more than a year, Babbage developed radically new ideas about how an automatic calculating machine should be organized. As a result, work on the original difference engine was never resumed.

## Difference Engines after Babbage

Although Babbage's difference engine was never finished, others were eventually built. In 1834 a Swedish publisher of technical journals, Georg Scheutz (1785-1873) read a description of Babbage's proposed engine, and decided to try to build one himself. In 1853, after many years of experiment and effort, Georg and his son Edvard completed a 15-digit difference engine working from constant fourth differences, which calculated and printed its results at the rate of two per minute. This was much slower than the rate of approximately ten per minute which Babbage was hoping to achieve, but there was the important difference that the Scheutz machine existed and worked, while Babbage's did not. Babbage did all that he could to publicize the Scheutz's achievement, and in 1859 a copy of the Scheutz engine was constructed by the engineering firm of Donkin & Co. for use by the Registrar-General in computing actuarial tables (Fig. 8). The original Scheutz engine went to the Dudley Observatory in Albany, New York, where apparently it was never used.

Another Swede, Martin Wiberg (1826-1905), built a redesigned version of the Scheutz engine, which he used for printing interest tables in 1860 and for other mathematical tables in 1875. A difference engine weighing more than a ton, designed by the American gear-cutting industrialist G.B. Grant, was exhibited at the Philadelphia Centennial Exposition of 1876. A machine to Grant's design was also sold to an American insurance company.

It is interesting to note that the existence of these nineteenth century difference engines did not, in fact, produce that revolution in the art of table-making which Babbage had so confidently predicted. It was not until 1929, when L.J. Comrie (1893-1950) of the British Nautical Almanac Office showed how certain commercially available accounting machines could be used as printing difference engines, that automatic tabulation by difference engine principles made any significant contribution to scientific calculation.

## Babbage's Analytical Engine

When work on the difference engine ceased in 1833, Babbage made use of what he believed was merely a temporary interruption to explore the consequences of some ideas which had been present in his mind from the beginning. One of these was the possibility of transferring groups of digits, possibly shifted by a power of ten, from the result register to the highest-order difference register, something which Babbage called "the engine eating its own tail". This led Babbage to consider rearranging the engine in circular form, the columns of digit wheels containing the differences being grouped around a central column of large gear wheels with which they could engage or disengage as required. These central gears would provide a means for transferring digits from any column to any other column.

Another variant of the difference engine principle which interested Babbage was the possibility of tabulating certain functions by the direct mechanical solution of an appropriate difference equation. For example, the function sin (x) satisfies a difference equation of the form

$$\Delta^2 \sin (x) = k \sin (x + 1)$$

where k is a constant.

Babbage therefore turned his attention to the design of suitable multiplying and dividing apparatus. He found that the speed of these operations would be heavily dependent on the speed of the carry mechanism, which he therefore improved still further. This mechanism, copies of which were needed at many places in the machine, was by now becoming disagreeably complicated. This led Babbage to the important idea of simplifying the machine by providing only a single carrying mechanism, which could be connected automatically via the central transfer gears to whatever pair of columns required it at a given moment. If this is done, the columns of digit wheels become mere passive number registers, relatively cheap and easy to manufacture. It was at this stage that Babbage made his major conceptual break-through: he realized that what had started out as a fixed-purpose difference engine could now be transformed into a general-purpose arithmetic calculator. Babbage immediately set to work on the design of such a machine, which was to consist of a "Store" containing "Variables" (columns of number wheels) and a separate "Mill" in which the arithmetic operations would be performed one at a time. Numbers would be transferred between the Store and the Mill by means of toothed racks.

To provide over-all control of the sequencing of operations in his new machine, Babbage at first proposed to use a rotating drum carrying studs

inserted by hand, similar to other such drums used for local control within the machine. Then, in June 1836, he made his second break-through by deciding to replace the control drum by a linked sequence of perforated cards similar to those used in the Jacquard loom: "It is easier to punch pasteboard than to screw on a multitude of studs. When once the formula has been made and verified, it need never be made again until worn out. The change from one formula to another, when both have been previously made, is done in a very short time. ... Every formula ever put into the engine will be preserved. The extent of the formulae is almost unbounded." Thus the Analytical Engine was born.

Babbage, in all his designs for the Analytical Engine, always supposed that the sequence of "Operation Cards" initiating the successive arithmetic operations would be physically distinct from the sequence of "Variable Cards" specifying the addresses of the operands and results. Distinct from both the Operation Cards and the Variable Cards would be the "Number Cards" containing numerical input data. Thus there would be at least three separate Jacquard-type mechanisms for reading cards. By 1837 Babbage's design had advanced sufficiently for him to draft a technical description entitled "On the Mathematical Powers of the Calculating Engine" (not published until 1973). Some of the features of the engine as described in this document are the following.

Storage for approximately 100 Variables each of 30 or 40 digits with sign.

Facilities for "backing" a sequence of Operation Cards (and presumably the corresponding Variable Cards), with associated counting apparatus allowing any specified number of repetitions of the sequence.

On-line punch for Number Cards, which could therefore be used for supplementary storage.

On-line printer.

On-line mechanical point plotter.

Off-line device for transferring data from Number Cards to embossed plates for the casting of type.

When writing of the Analytical Engine twenty-seven years later in his autobiography, Babbage explains how the action of backing the cards can be made to depend on the change of sign of the number on one of the Variables. He also writes: "Mechanical means have been provided for backing or advancing the operation cards to any extent". This strongly suggests that the Analytical Engine would have had the equivalent of a modern conditional branching facility, though Babbage could hardly have appreciated the full power of this innovation.

From 1834 to 1846 Babbage worked on the design of the Analytical Engine without making any attempt to build it. He took the subject up again in 1857 and worked on it intermittently until his death in 1871, apparently in the hope that something might be constructed. Shortly before he died he made arrangements for the assembly of a small demonstration section of the engine, consisting of two Variable columns, a column containing the carrying mechanism, part of the printing mechanism and the associated transfer racks (Fig. 11). This was completed after his death by his son, H.P. Babbage.

If Babbage's engine had been completed, the times for the arithmetic operations would probably have been approximately as follows:

| | |
|---|---|
| Addition and subtraction | 2 sec (possibly appreciably longer for the first term of a sum). |
| Multiplication and division | 1 min. |

As to the feasibility of Babbage's design, we may quote the opinion of Prof. M.V. Wilkes: "At best it would have been very temperamental and subject to frequent failure. It would also have disillusioned Babbage about machines being incapable of making mistakes. However, the same could be said of the early electronic computers that were built 100 years later".

## Programming the Analytical Engine

Babbage himself never published a detailed account of his Analytical Engine. However in 1842 a twenty-five page description written in French was published in Geneva by a young Italian military engineer, L.F. Menabrea (1809-1896), afterwards prime minister of Italy, who had been one of a group of mathematicians to whom Babbage had explained his ideas during a visit to Turin in 1840. In 1843 Menabrea's description was translated into English by the Countess of Lovelace (1815-1852), daughter of the poet Lord Byron. Lady Lovelace had studied mathematics and was a close friend and disciple of Babbage. Her translation was accompanied by supplementary notes written by herself, amounting to two and a half times the length of Menabrea's article, and describing in detail the capabilities, limitations and possible applications of the Analytical Engine.

In order to illustrate the way in which Babbage's engine could be used, Lady Lovelace provided a program (in the form of the table shown in Fig. 12) for the iterative calculation of the Bernoulli numbers. The notation used in this program is essentially a three-address code, with the addition of an upper left superscript to distinguish the successive values assumed by each Variable. Thus, "${}^2V_6 - {}^1V_1 = {}^3V_6$" (Operation 17 in the table) is merely a particular case of "$V_6 - V_1 \rightarrow V_6$". Read-out from the store is destructive, leaving zero on any Variable which has been read; the convention is that any Variable which is to receive a copy of its initial value, or of any result, must be listed explicitly in the fifth column of the table. Having disposed of these preliminaries, we can now discuss the program itself.

The Bernoulli numbers $B_1$, $B_2$, $B_3$, ... ($B_2$, $B_4$, $B_6$, ... in modern notation) are to be stored, as they are calculated, on Variables $V_{21}$, $V_{22}$, $V_{23}$, ..., respectively. Calculation is by means of the recurrence relation

$$A_0 + A_1B_1 + A_3B_3 + \dots + A_{2n-3}B_{2n-3} + B_{2n-1} = 0 ,$$

where

$$A_0 = -\frac{1}{2}\,\frac{2n-1}{2n+1} ,$$

$$A_1 = n ,$$

$$A_{2r-1} = \frac{2n(2n-1)\ \dots\ (2n-2r+2)}{(2r)!}$$

$$= \frac{(2n-2r+3)(2n-2r+2)}{(2r-1)(2r)}\,A_{2r-3},\quad r = 2,3,4,\dots .$$

Operations 13 to 23 comprise the inner loop of the program, causing a new term $A_{2r-1}B_{2r-1}$ to be added to the sum which is being accumulated on $V_{13}$. On entry to this loop,

$$V_6 = 2n - 2r + 4 ,$$

$$V_7 = 2r - 2 .$$

The program illustrates the particular case $n = 4$, $r = 2$. Throughout the program $V_{10}$ is used as an iteration counter, starting at value n and decreasing by unity each time a Bernoulli number is calculated. It is necessary to interpret the horizontal lines following Operations 7 and 12 as meaning "Branch to Operation 24 if $V_{10} = 0$", and to interpret the blank line following Operation 23 as meaning "Branch to Operation 13 if $V_{10} \neq 0$". Following Operation 25, the program needs an instruction "Branch to Operation 1 if $V_3 \neq N$", where $B_{2N-1}$ is the last Bernoulli number required.

It is clearly explained in the notes accompanying the program that the only operations which are not invariant are those numbered 21 and 24. Lady Lovelace then continues, "But as these variations follow the same law at each repetition (Operation 21 always requiring its factor from a column <u>one</u> in advance of that which it used the previous time, and Operation 24 always putting its result on the column <u>one</u> in advance of that which received the previous result), they are easily provided for in arranging the recurring group (or cycle) of Variable-cards". It is interesting to speculate whether Lady Lovelace and Babbage -- if they had considered carefully the implications of this last remark -- might not have been led to invent some mechanical equivalent of an index register! As it is, it is quite possible that Lady Lovelace realized that the set of variable cards associated with Operations 13 to 21 would not have to be copied out more than N - 1 times, one of the addresses associated with Operation 24 being increased by unity in each copy. This is a lot better than the $\frac{1}{2}N$ (N - 1) copies of these cards which would be needed if the program were written out explicitly without any use of conditional branching.

Lady Lovelace's program, with its indexed variables and doubly nested loops, was as far ahead of its time as the imaginary computer for which it was written. More than a hundred years were destined to pass before anyone would again be writing a program of this degree of sophistication.

## Punched Card Machinery

In 1880, Dr. J.S. Billings, Director of Vital Statistics for the United States census of that year, mentioned to a young graduate engineer, Herman Hollerith (1860-1929), who had recently joined the Census Office that there ought to be some mechanical way of tabulating the census returns. Billings suggested that the information relating to each individual might be recorded in some machine-readable form on a single card. Hollerith followed up this idea, and during the next four years he developed and patented a tabulating system which used rectangular cards on which information could be recorded in the form of circular holes. Hollerith's system was adopted for the analysis of data from the next census, that of 1890, and was highly successful.

Figure 13 shows the cover of the Scientific American for the 30th August 1890, illustrating Hollerith's system. Cards which had been perforated by means of a pantograph punch were placed one at a time in a card-reading press. When the lever of this press was pulled down, spring-loaded pins could penetrate the holes, completing an electric circuit through one or more of the clock-like counters set in a frame above the press. A sorting box was also available; completion of the circuit through a hole of the card would cause the lid of one of the compartments to spring open; the operator would then take the card out of the reading press, drop it into the compartment, and close the lid by hand. It is interesting to note that, even in this 1890 system, Hollerith was already using relay circuits to select specified combinations of holes for counting (e.g. "male and aged 10 to 14"), as well as to check for logically incompatible combinations of holes (e.g. "married and aged 10 to 14").

In 1896 Hollerith set up his own company to manufacture and lease -- but not to sell -- his machines. This company was later absorbed into the larger company which eventually became the International Business Machines Corporation. The evolution of the punched card machines which we know today was relatively slow; automatic card-feeding was introduced around 1900, and the printing tabulator in 1913 -- by the rival Powers Accounting Machine Company. Also in 1913 came the first accumulating tabulator. This exploited the punched card as a <u>numerical</u> data record; until then the card had been regarded primarily as a two-dimensional medium for recording descriptive characteristics for subsequent sorting and counting.

Until the arrival of the electronic computer, automatic calculation meant essentially punched card calculation. But even after the introduction of the multiplying punch in 1931 the relative inflexibility and high rental charge of punched card machines made them unsuitable for general-purpose scientific work. An exception was the successful use of these machines for astronomical calculation, particularly by L.J. Comrie in England and W.J. Eckert at Columbia University, whose equipment was donated by IBM. The Columbia bureau is also noteworthy for the electromechanical programming device which Eckert constructed to coordinate the action of several standard punched card machines.

## From 1900 to approximately 1935

The most important computational event of this period was probably the invention in 1930 of the differential analyser by Vannevar Bush (1890-1974) at M.I.T. This mechanical analogue device for the solution of systems of ordinary differential equations made it possible for engineers and others to obtain, in hours rather than weeks, solutions of adequate accuracy to many important problems of research and design. However, our concern here is with digital rather than analogue computers. Although much of the mechanical, electrical and electronic technology used in the later automatic computers was developed during this period, no one seems to have made any serious attempt to construct such a machine. There were, however, a number of interesting proposals and experiments, some of which we shall now describe.

In 1909 a Dublin accountant, P.E. Ludgate (1883-1922) published a paper outlining a plan for the construction of a compact analytical engine, functionally very similar to that of Babbage, with storage for 192 numbers of 20 digits. The numbers were to be represented by the transverse displacements of pegs held in small "shuttles" which could be withdrawn from addressed compartments in a rotatable cylinder. Arithmetic was to be performed mechanically, using an ingenious system of integer logarithms. Important ideas in Ludgate's proposal

include the following: (1) sequence control by means of a perforated paper roll in which each row of perforations represented a single "instruction" in the modern sense of the word, consisting of a function code and two addresses; (2) provision for conditional branching during program execution -- but without any detailed explanation; (3) provision for the machine to punch numerical data into number-rolls for subsequent input; (4) automatic printing of results; and (5) division by means of a subroutine roll held on a separate reader. Ludgate explains that his machine would be simpler and much smaller than that of Babbage.

The Spanish engineer and inventor Torres y Quevedo (1852-1936) was particularly interested in questions of automatic control and automatic calculation. In a 1914 paper entitled "Essais sur l'automatique ..." he shows how, at least in principle, a flexible automatic calculating machine could be constructed using electromechanical devices. The machine could be programmed by means of a rotating drum carrying appropriately positioned conducting studs; decimal number storage could be provided by sliding strips; and data could be read into the machine from a sliding plate carrying contacts. There was also to be provision for testing intermediate results for zero; but it seems that the subsequent action would depend on the wiring of the machine. It was proposed that the machine should use floating-point decimal arithmetic, each number being assigned a two-digit decimal exponent. This must be one of the earliest references to the use of floating-point arithmetic in calculating machinery.

During the 1930's several inventors proposed that binary arithmetic might be used in calculating devices because of its logical simplicity. For example, a 1932 patent application by R.L.A. Valtat describes a mechanical, binary, hand-operated calculator with a decimal keyboard for input, and with decimal output. Conversion between decimal and binary was to be automatic. In 1936 an English actuary, E.W. Phillips (1892-1968), described a mechanical binary multiplier very similar to that of Valtat (to whom he refers), and then goes on to propose a multiplying machine in which sequences of pairs of numbers would be read photo-electrically from two perforated paper rolls. The binary multiplications were to be carried out by photocells and high-speed relays. Phillips suggested that, if such machines became availabe, actuaries should use octal (scale of 8) numbers for all records and tables! Many years later Phillips stated that he had in fact intended his binary multiplier to be electronic, but that he had deleted all references to vacuum tubes because patent applications had not been filed.

Also in 1936, Louis Couffignal published a brief and imprecise description of a binary calculator using relays, operating on signed numbers, and incorporating some kind of analogue-to-digital and digital-to-analogue mechanical input and output. The purpose of the device was withheld for unspecified reasons of secrecy. Later, in his 1938 doctoral thesis, Couffignal put forward a proposal for a general-purpose automatic binary calculator in which the control information (i.e. the program), input data, internal tables, and (if necessary) some of the intermediate results, would be perforated into rolls or strips of cardboard or thin metal. The roll containing the input data was to be prepared using an off-line perforator incorporating a decimal-to-binary converter. Output results were to be converted automatically to decimal form and then printed by an on-line typewriter. The machine's internal binary storage was to be mechanical. Relays and special electromechanical mechanism were to be used for the arithmetic operations. The estimated time for multiplication would be about three seconds. Table-searching and interpolation could take place on associated independent reading mechanisms, subroutine fashion. Couffignal also suggested that a special punching and reading mechanism could be constructed to handle perforated two-dimensional arrays of numbers representing intermediate results in matrix calculations.

By the 1930's the art of designing logically complicated relay circuits had been brought to an advanced level by telephone switching engineers. However, the price of telephone relays was too high for these to be used indiscriminately as logical elements by anyone not working for a PTT organization.

During the 1920's vacuum tube technology had advanced rapidly and had become the foundation for a major industry. As a result, knowledge of electronic circuit design methods became widespread. Two developments in electronics which were of importance for the future history of the electronic computer were the invention of the flip-flop by W.H. Eccles and F.W. Jordan in 1919, and the development of the first electronic counting circuits for ionization chambers (using thyratron discharge tubes) by C.E. Wynn-Williams in 1931. From that time onwards, it is likely that most electronic engineers, if asked, would have agreed that all-electronic arithmetic was a practical possibility -- but they would probably have added a warning about high cost and low reliability.

A history of computing should make some reference to the difficult, but delightfully written, 1936 paper by A.M. Turing (1912-1954) entitled "On computable numbers, with an application to the Entscheidungsproblem". Turing defines a computable number as one which can be printed by a certain abstract computing machine which he describes. Such a number is completely defined by a finite sequence of symbols placed in the infinite working store of the machine before it is set into action, this sequence being the equivalent of a program. It is unlikely that Turing's purely theoretical paper was read and understood at the time by anyone except a small number of specialists in mathematical logic -- amongst whom was certainly John von Neumann.

## The Zuse Z3 Computer

The use of a perforated paper band (or its equivalent, a linked sequence of perforated cards) as a control device had been well known to engineers for a long time. Some examples are the Jacquard loom, the mechanical piano-player, and -- later -- the Linotype composing machine. As we have seen, this method of control had been proposed for use in automatic calculating machines by Babbage, Ludgate, Couffignal, and probably others. It is convenient to refer to such a machine as "externally programmed".

From about 1935 onwards, Konrad Zuse, a structural engineer working in Berlin, began to construct a series of tape-controlled electromechanical computers, the tape being in the form of perforated 35 mm cinema film. In a patent application submitted in 1936 Zuse described a computer of this kind having the following features: floating-point binary arithmetic, a multiword relay store, an arithmetic unit based on an electromechanical binary adder, and relay logic. However, the computer which Zuse was actually building at this time used an ingenious mechanical binary store rather than the more

expensive relay store mentioned in his patent application. (Incidentally, Zuse's American patent application was rejected with a reference to the prior invention by Babbage.)

Zuse's most remarkable achievement was the completion in 1941 of what he afterwards called the Z3 computer. This was a tape-controlled (i.e. film-controlled) all-relay binary computer having relay storage for 64 numbers of 22 bits: 7 bits for a signed binary exponent and 15 bits for a signed binary mantissa. The arithmetic unit used parallel arithmetic. The instruction repertoire included a square root instruction, and there were separate fetch and store instructions for referencing the memory, but there was no provision for conditional branching in program execution. Numerical input was from a four-digit decimal keyboard, and output was by means of a four-digit panel of lamps. There was automatic decimal-to-binary and binary-to-decimal conversion. Approximate operation times were: addition 0.3 seconds, multiplication 4.5 seconds.

Some very reliable special-purpose relay computers were built by G.R. Stibitz of the Bell Telephone Laboratories, starting in 1939; but it was not until 1946 that Stibitz constructed a general-purpose relay computer. Therefore, as far as is known, Zuse's Z3 was the world's first working general-purpose programmed computer. It was used for various experimental calculations, but not as a "production" machine, and was eventually destroyed during an air-raid in 1944. Figure 14 shows a reconstruction of this computer.

## The Harvard Mark 1 Calculator

The first automatic computer in the United States originated from a proposal put forward in 1937 by H.H. Aiken (1900-1973). Aiken, who was at that time an instructor in the Department of Physics at Harvard University, approached various manufacturers of calculating machines to explain his ideas on how a flexible externally programmed calculator could be built. He succeeded in interesting IBM, who assigned some of their best engineers to collaborate with Aiken in the detailed design of the machine. Construction began in 1939 and was finished in 1944, when the giant IBM Automatic Sequence Controlled Calculator (usually known as the Harvard Mark 1 Calculator) was presented to Harvard University by Thomas J. Watson as a gift from IBM (Fig. 15).

The Mark 1 was constructed as far as possible from standard IBM components. Internal storage took the form of 72 accumulators made up of counter wheels, each accumulator holding 23 decimal digits. There was also "read-only" storage, in the form of hand-switches, for 60 numbers of 24 digits. Digits were added or subtracted in parallel by connecting and disconnecting the counter wheels from a rotating shaft at appropriate clock times by means of magnetic clutches. Basic operation times were: addition and subtraction 0.3 seconds, full multiplication 6 seconds, division approximately 11 seconds. Special units were provided for computing the functions $\log_{10} x$, $10^x$, and sin x. Over-all control of the machine was from a 24-hole paper tape, using a complicated code to specify operations and addresses. There were also three subsidiary interpolation units on which tapes containing values of special functions could be placed. Input and output were from standard punched cards, supplemented by two on-line electric typewriters. Punched cards could therefore be used as a supplementary off-line memory.

The principal limitation of the Mark 1 was the absence of any conditional branching facility. The machine could not transfer control from one part of its main control tape to any other part, either forwards or backwards. Thus all programs, no matter how repetitive, had to be written out explicitly. For example, the solution of a system of n linear equations in n unknowns required a separate tape for each value of n. For n = 10 the tape consisted of nearly 5000 lines of coding, and the running time was approximately 45 minutes.

## Electronic Computation

Prior to World War II there seems to be no evidence of any serious attempt to construct an entirely electronic computer. One place where the idea had certainly been considered, and where preliminary investigations were under way, was M.I.T. The extent to which this computing know-how was generally diffused at M.I.T. is shown by P.J. Crawford's interesting Bachelor of Science thesis (1939) entitled "Instrumental Analysis in Matrix Algebra". In this thesis Crawford proposes the construction of a digital electronic calculator whose basic operation is to be the multiplication of matrices. The matrices, of order up to 50 by 50, were to be read photoelectrically from arrays of perforated strips placed side by side. Numbers were to be represented in floating-point decimal form with a two-digit decimal exponent and an eight-digit mantissa. Over-all control was to be from a perforated tape. An interesting feature of the proposal is the suggestion that results could be printed by a high-speed "on-the-fly" printer. Crawford does not give any information about how the machine might be constructed, but merely states: "Many of the details of a calculating system similar to that suggested by the above have been considered by Bush [in unpublished Memoranda], and it is not proposed that they be reproduced here". Unfortunately, these memoranda are still unpublished.

During the war Crawford drew up a proposal for an anti-aircraft fire control system using 14-bit binary arithmetic for predicting the position of the target. This was the subject of his 1942 Master's thesis. Numerical tables required by the system were to be made available on optical transparencies, and temporary storage for numbers was to be provided on a magnetic (metal?) tape forming the circumference of a rotating disc. An anti-aircraft fire control system using digital arithmetic was also designed during the war by the Radio Corporation of America (RCA), and there seems to have been some collaboration between Crawford and RCA at this time.

## The Atanasoff Computer

We now come to a computer project which, more or less accidentally, initiated the chain of events leading to the computer as we know it today. Paradoxically, this project involved the construction of a calculator which was neither general-purpose nor entirely electronic. This device was the digital simultaneous equation solver designed by J.V. Atanasoff at Iowa State University, and constructed by Atanasoff and C.E. Berry during the period 1938 to 1942, in which year work ceased because both Atanasoff and Berry transferred to other war work.

The Atanasoff computer was intended to solve systems of linear equations of order up to 29 by elimination, using fixed-point 50-bit binary arithmetic. Two rows at a time of the matrix were held on two storage drums rotating on a common shaft at a rate of one revolution per second. Binary digits

were represented by the state of charge of capacitors contained in the drums, the capacitors being connected to 30 rings of conducting studs on the surface of the drum, the 50 studs of each ring (representing one coefficient) passing in succession under a corresponding fixed brush which sensed the potential of each stud and restored the charge. The serial arithmetic unit was ingeniously arranged so that the elimination operations for a complete row of the matrix were completed in one multiplication time. Using the index k to identify the pivotal row, held on one of the drums, and the index m to identify the row being currently processed, held on the other drum, the process performed during these revolutions was

$$a'_{mj} \leftarrow a_{mj} - \left(\frac{a_{m1}}{a_{k1}}\right) a_{kj}, \quad j = 1, 2, \ldots, n .$$

The new row of elements $a'_{mj}$ was to be punched by spark discharges onto a single non-standard 30 by 50 binary card as input for the next run through the matrix. Input was to be from standard decimal punched cards. The final result vector was to be displayed on decimal dials, presumably one element at a time. Allowing for the time taken by the automatic decimal-to-binary conversion of the input matrix, the total time for solving a system of 29 equations would probably have been at least 14 hours. The time for solving a system of this size with ordinary desk machines would have been several weeks. When work on the calculator was abandoned in 1942, the electronic arithmetic unit was working, but the special punched card devices were still under development.

## The ENIAC

The importance of the Atanasoff machine for the history of computing is that it came to the notice of J.W. Mauchly, who in 1940 was teaching physics at a small private college in Pennsylvania. Mauchly had been experimenting with computing devices for some years, and was also familiar with punched card equipment. In June 1941 Mauchly arranged to spend several days with Atanasoff at Iowa State University, and was impressed by the advantages of digital methods over analogue.

In 1942 Mauchly found himself on the teaching staff of the Moore School of Electrical Engineering of the University of Pennsylvania. The Moore School had a contract with the United States army for the production of ballistic tables, which they were at that time calculating with the aid of a differential analyser and a large group of desk machine operators. Also on the staff of the Moore School was a young electrical engineer, J.P. Eckert. Mauchly discussed with Eckert the possibility of using a completely electronic digital computing device to solve arithmetically the finite difference approximations to the external ballistic equation. As a result of these discussions, Mauchly drafted a memorandum in August 1942 describing in fairly general terms a computing system of this kind. In March of the following year, 1943, the mathematician H.H. Goldstine who was in charge of the Moore School computing project on behalf of the army, urged Eckert and Mauchly to make a formal request to the army authorities for funds to build their proposed machine. This they did, and the request was officially approved within two months of being submitted.

Work on the ENIAC (Electronic Numerical Integrator and Computer) began in mid-1943. It was not only the world's first automatic electronic computer; it was also far and away the world's fastest and largest. When completed, it contained more than 18000 vacuum tubes and consumed more than 150 kilowatts. Internal storage in the ENIAC took the form of 20 accumulators each holding 10 decimal digits. Each digit was held in a ring counter requiring 20 tubes, and the total number of tubes in each accumulator was 550. The ENIAC also had three "read-only" function tables, each capable of holding 104 twelve-digit numbers set up on hand switches.

Although the ENIAC was intended primarily for the solution of differential equations, and although it did not operate with a "program" in the modern sense of the word, it was nevertheless a general-purpose computer. The sequencing of basic chains of arithmetical operations was determined by the connections established between the units by pluggable coaxial cables, and by the settings of switches on the units themselves. Control of loop repetitions and hierarchical over-all control were provided by electronic stepping switches forming a Master Programmer unit. Conditional transfers of control depending on intermediate results were a standard feature of the control system. (One might call the control system of the ENIAC "programming by configuration".) Arithmetically, the machine operated rather like an electronic version of the Harvard Mark 1 -- of which Eckert and Mauchly knew nothing at the time. That is, it operated by counting, in parallel, digit pulses generated by a central clock, using electronic ring counters instead of counting wheels. The operation times were: addition and subtraction 0.2 milliseconds, multiplication 2.8 milliseconds, division approximately 20 milliseconds. The ENIAC was thus more than a thousand times faster than the Harvard Mark 1. Input and output was by means of standard punched cards, using buffered read and punch stations to avoid delaying the electronic equipment.

The ENIAC (Fig. 16) was not completed until December 1945, four months after the end of the war in the Pacific. Its reliability never approached that of the electromechanical Mark 1, but the fact that it worked at all was a remarkable tribute to the skill of Eckert and his collaborators. However, the limitations of the machine's inflexible control system were always troublesome; the time required for changing from one calculation to another, including planning, set-up time and debugging, could amount to several weeks.

## The EDVAC proposal

The ENIAC was designed to meet wartime needs. Existing vacuum tube technology was employed, as conservatively as possible, in conjunction with straightforward arithmetic methods such as pulse counting, to provide a machine which would be automatic in action and extremely fast, but not necessarily economical in the number of vacuum tubes used. The design was frozen before the end of 1943, and by early 1944 Eckert and Mauchly were already thinking about how to build a less wasteful successor to the ENIAC, whose principal drawbacks were already recognized: namely the inadequate amount of high-speed storage, and the awkwardness of the method for changing from one calculation to another.

Before describing the gradual evolution of ideas at the Moore School during 1944, it is useful to summarize the essential features of what we would now call a stored program computer, none of which was present in the ENIAC. These are:

1) The concept of control by program, where the program consists of instructions coded in digital form.

2) The use of a single high-speed store to hold both the program and the numerical data on which it operates.

3) The ability of the program to modify its own instructions.

4) The provision of a special instruction for conditional branching -- or at least the existence of means by which the machine can simulate the action of such an instruction.

It is items 3 and 4 of this list which give the modern computer its remarkable powers; and this remains true even if the modification of instructions is performed by means of an index register instead of being explicitly programmed.

Let us now return to 1944. By February of that year Eckert and Mauchly were discussing a "magnetic computer" in which pulses representing numerical data were to be recorded on the edge of a magnetic disc, an idea which was possibly taken over from Crawford. Then, later in the year, came the important invention by Eckert of the closed mercury delay line as a storage device. Suddenly high-speed storage became, in principle, relatively cheap and plentiful.

In the summer of 1944, Goldstine got into conversation with the Hungarian-born mathematician John von Neumann (1903-1957) while waiting for a train to Philadelphia. Von Neumann, who was already a consultant to several military development programs, including the highly secret atomic bomb project, was extremely interested by what Goldstine told him about the ENIAC, and quickly managed to have himself appointed as a consultant to the Moore School. From then on, he would come down to the Moore School from time to time for discussions about the new computer which was to succeed the ENIAC. No records seem to have been kept of these discussions, so it is impossible to say who thought of what.

By early September 1944 the stored program idea had been clearly formulated. Thus, in a letter to P.N. Gillon dated 2nd September 1944, Goldstine writes: "To remedy this disparity [between set-up time and computing time] we propose a centralized programming device in which the program routine is stored in coded form in the same type storage devices suggested above". At exactly what stage the importance of program modification and conditional branching were recognized it is not possible to say. What we do know is that by the spring of 1945 von Neumann was writing experimental programs for an imaginary computer possessing these features, and that the structure of this computer, to the called the EDVAC (Electronic Discrete Variable Computer), was the subject of a now famous mimeographed report issued by the Moore School on the 30th June 1945 bearing the title "First Draft of a Report on the EDVAC by John von Neumann".

It is fairly clear that this report by von Neumann was not intended for publication. It was probably exactly what its title stated -- a first draft, presumably intended as a basis for discussion with von Neumann's colleagues in the EDVAC development. Much of the report is concerned with arithmetical circuitry, treated at the level of idealized AND and OR gates. There is also a careful discussion of the trade-offs involved in choosing word size, delay-line length, and total storage capacity. The proposed machine is binary and serial, with one instruction per word, and with a suggested word-length of 32 bits. Instructions which refer to storage are of one-address form. The instruction repertoire includes instructions which allow a program to modify itself and to execute conditional jumps, but there is no discussion of how these facilities would be used in programming the machine; perhaps von Neumann considered this to be self-evident. The instruction code is, by modern standards, rather clumsy. In particular, a conditional transfer of control requires the execution of four instructions, one of which has to do quite a lot of work. The fact that even von Neumann did not arrive immediately at the idea of executing a conditional jump by means of a single instruction suggests that this concept may be less obvious than we think.

## From 1946 to 1949

In 1946 Eckert and Mauchly left the Moore School to set up their own computer company. In the same year von Neumann and Goldstine moved to the Institute of Advanced Studies in Princeton, where, during the period 1946 to 1948, they published in collaboration with A.W. Burks a series of detailed reports on the design and programming of stored program computers -- none of which as yet existed.

An important stimulus to the actual construction of the new kind of computer was provided by a series of lectures given at the Moore School in the summer of 1946 by Eckert, Mauchly, von Neumann, Burks, Goldstine and others. Those of the twenty-eight invited participants (several from Europe) who could obtain funds set to work to build their own version of an EDVAC-type computer. This computer-building race was won by M.V. Wilkes of Cambridge University, whose 512-word EDSAC (Electronic Delay Storage Automatic Computer), complete with paper tape input and on-line teleprinter output, was working in May 1949 (Fig. 17).

There remains one important development to be noted before closing this historical record: namely the invention of the index register. This invention eliminated -- at least as far as most arithmetical programs are concerned -- the awkward need for programs to modify their own instructions, something which was originally considered essential for the effective use of a stored program computer. The idea originated at Manchester University in a simple experimental computer model (lacking input and output devices), built by F.C. Williams and T. Kilburn, which was working in April 1949. This machine had two special registers, one of which normally contained zero, while the other contained a base-address which could be added to suitable tagged instructions to provide for the displacement of subroutines. That, at least, was the original intention, but it was very quickly realized that one can do a lot more with such registers than merely displace subroutines!

## Conclusion

The last word on the stored program computer can perhaps be left to Konrad Zuse. When talking recently about his wartime work on computers, Zuse pointed out that, as soon as it was recognized that a program could be represented in binary form, it clearly became possible for a program to treat itself as data; and then he added: "I felt that to do that was to enter into a contract with the devil".

## Acknowledgements

I should like to thank the persons and institutions who have supplied the accompanying illustrations and granted permission to reproduce them.

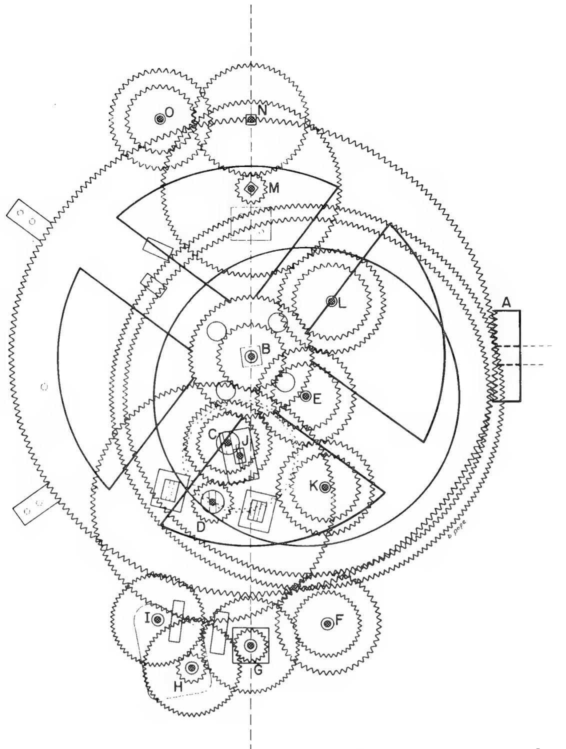
## Suggested books

1) The origins of digital computers: selected papers, by B. Randell (Springer-Verlag, Berlin, 1973).

   Reprints of source documents in computer history from 1837 to 1949. Each section is preceded by a clear and concise historical summary. There is a very full bibliography, with short comments on each item.

2) A computer perspective, by the Office of Charles and Ray Eames (Harvard University Press, Cambridge, Mass., 1973). (Also available with paper covers.)

   A well-planned book of photographs covering not only computing machines, but also the social and technical background against which they evolved.




D. de Solla Price & Amer. Phil. Soc.

Fig. 1 The Antikythera calendrical mechanism (ca. 90 B.C.). Probable arrangement of the gearing.

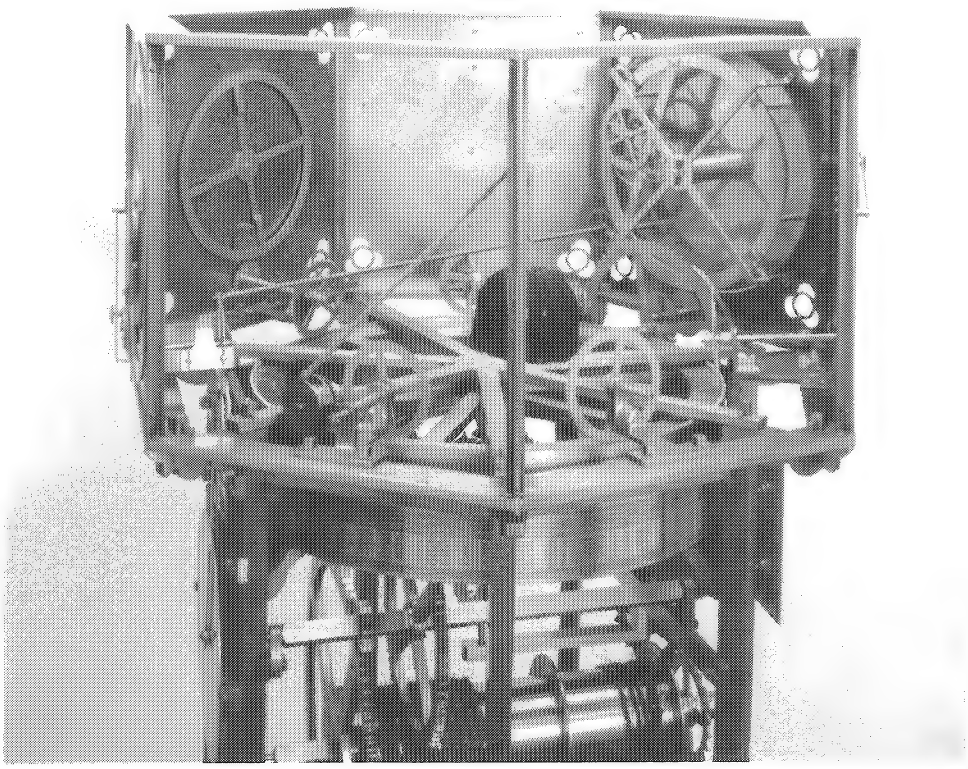


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Fig. 2 De Dondi's astronomical clock (1364). A portion of a modern reconstruction.

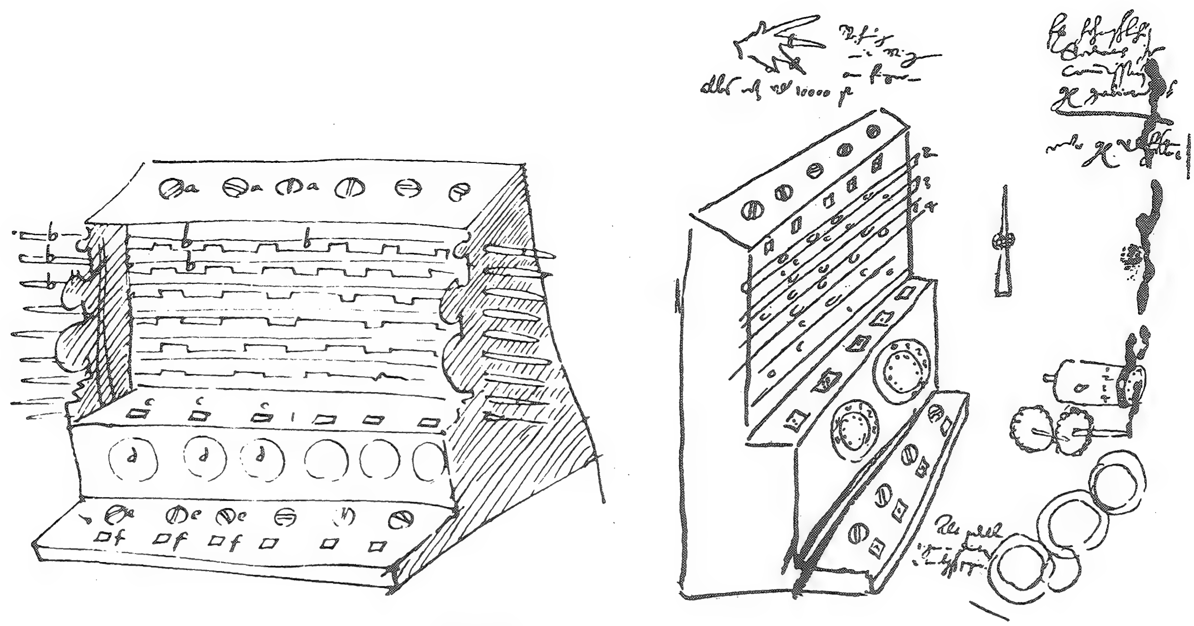

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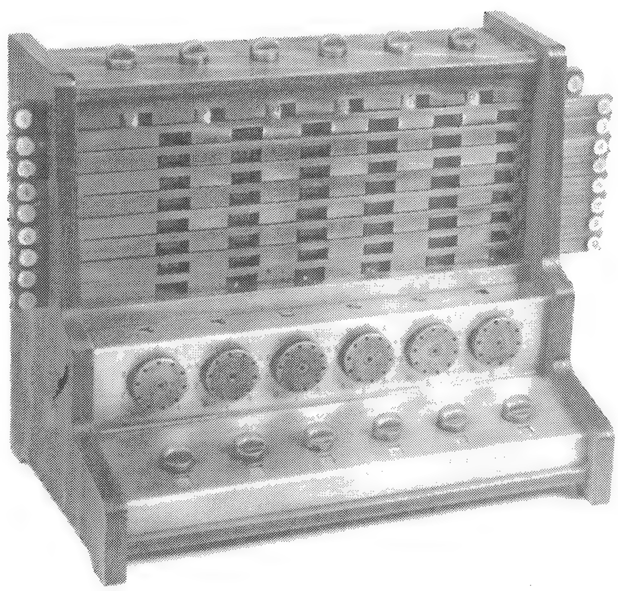
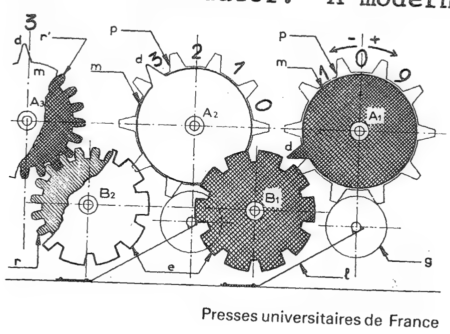
Fig. 3 Two sketches by Wilhelm Schickard of his calculator (1623)



IBM Deutschland

Fig. 4 Schickard's calculator. A modern reconstruction.




Presses universitaires de France

Fig. 5 Schickard's calculator. Possible arrangement of the carry mechanism.


IBM Deutschland

Fig. 6 Charles Babbage (ca. 1828)

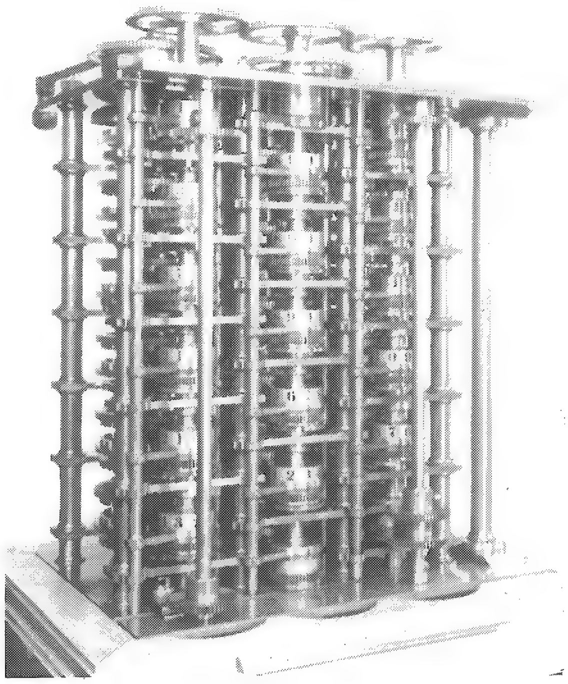


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Fig. 7 Babbage's difference engine (never completed). A small portion assembled in 1832.

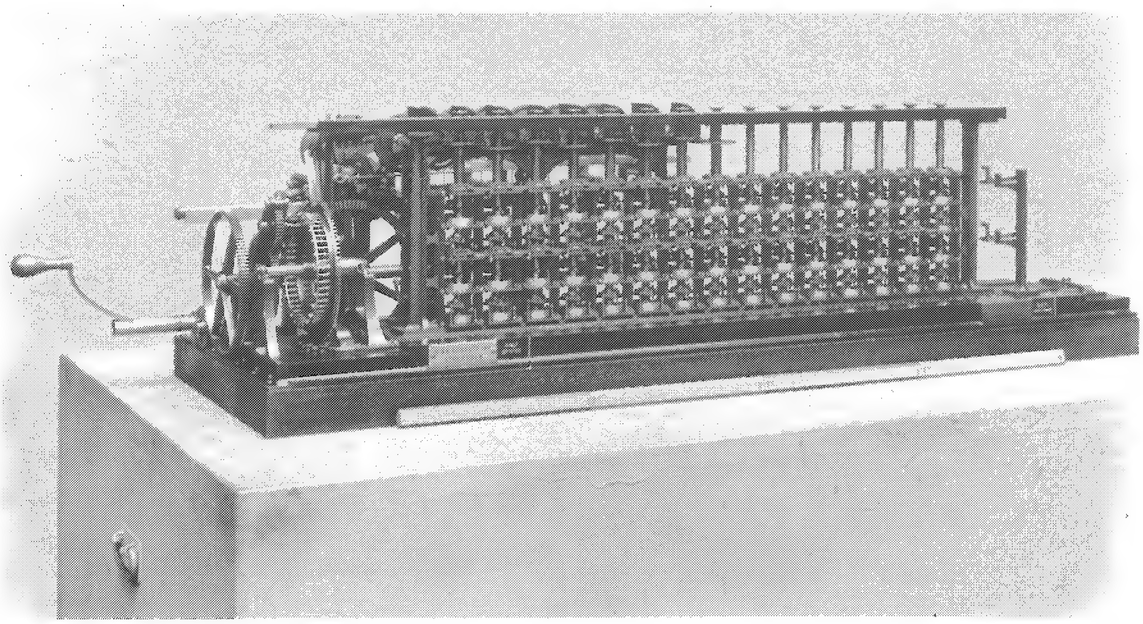

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Fig. 8 Scheutz's difference engine. A copy built in 1859.

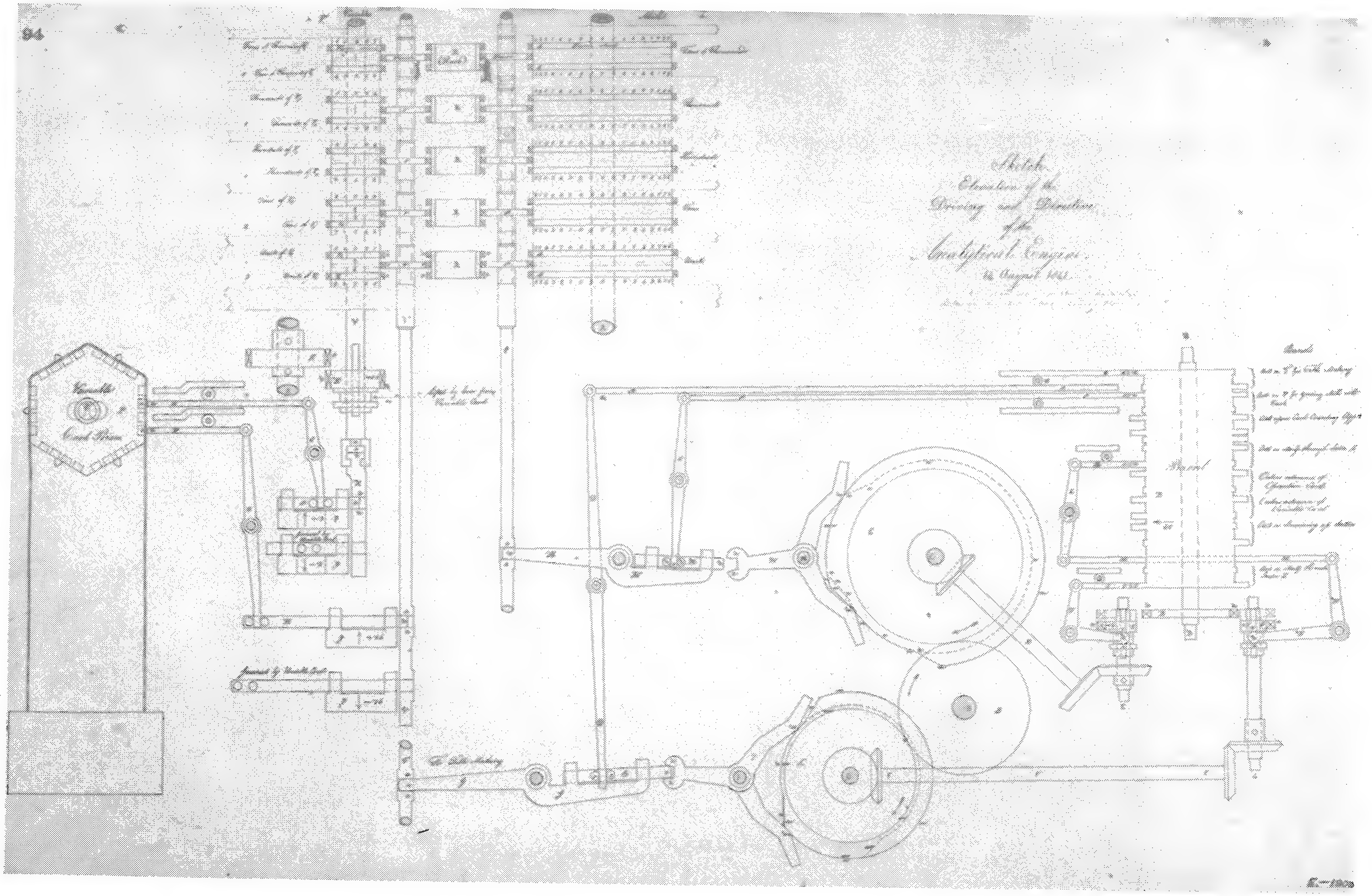

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Fig. 9   Babbage's Analytical Engine.  Working drawing (1841).

B.V. Bowden

Fig. 10 Ada Augusta, Countess of Lovelace

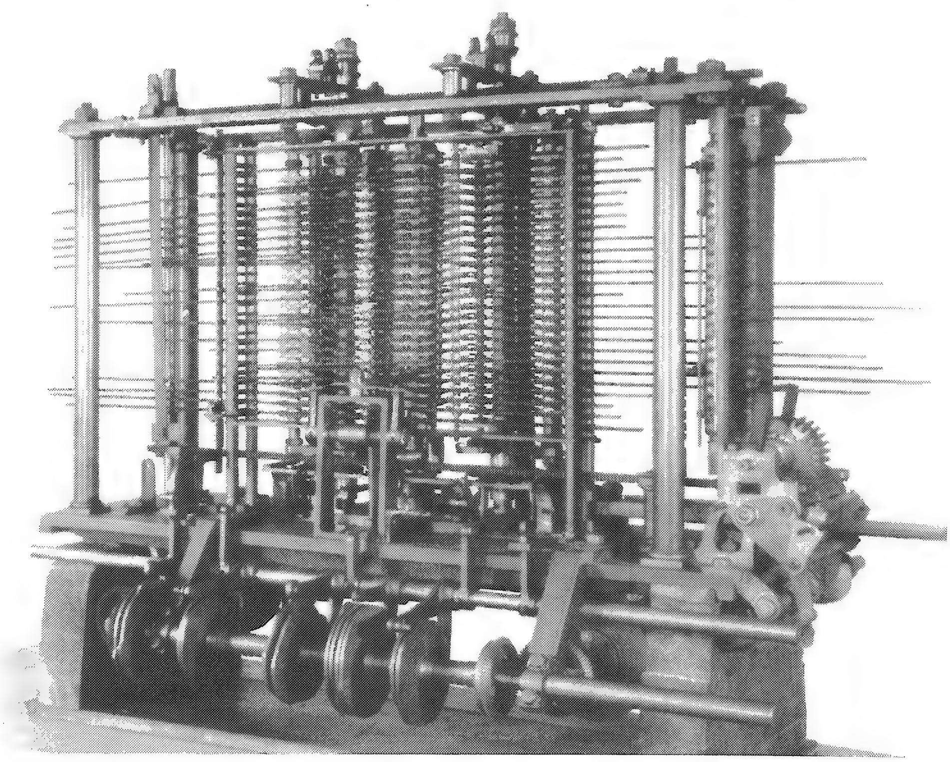

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Fig. 11 Babbage's Analytical Engine (never constructed). Fragments assembled ca. 1872.

Diagram for the computation by the Engine of the Numbers of Bernoulli. See Note G. (page 722 *et seq.*)

| Number of Operation. | Nature of Operation. | Variables acted upon. | Variables receiving results. | Indication of change in the value on any Variable. | Statement of Results. | Data. $^1V_1$ 0001 [1] | $^1V_2$ 0002 [2] | $^1V_3$ 0004 [n] | Working Variables. $^0V_4$ 0000 | $^0V_5$ 0000 | $^0V_6$ 0000 | $^0V_7$ 0000 | $^0V_8$ 0000 | $^0V_9$ 0000 | $^0V_{10}$ 0000 | $^0V_{11}$ 0000 | $^0V_{12}$ 0000 | $^0V_{13}$ 0000 ......... | Result Variables. $^1V_{21}$ $B_1$ in a decimal fraction. [$B_1$] | $^1V_{22}$ $B_3$ in a decimal fraction. [$B_3$] | $^1V_{23}$ $B_5$ in a decimal fraction. [$B_5$] | $^0V_{24}$ ... 0000 [$B_7$] |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | × | $^1V_2 \times {}^1V_3$ | $^1V_4, {}^1V_5, {}^1V_6$ | $^1V_2 = {}^1V_2$<br>$^1V_3 = {}^1V_3$ | $= 2n$ | ... | 2 | $n$ | $2n$ | $2n$ | $2n$ | | | | | | | | | | | |
| 2 | − | $^1V_4 - {}^1V_1$ | $^2V_4$ | $^1V_4 = {}^2V_4$<br>$^1V_1 = {}^1V_1$ | $= 2n-1$ | 1 | ... | ... | $2n-1$ | | | | | | | | | | | | | |
| 3 | + | $^1V_5 + {}^1V_1$ | $^2V_5$ | $^1V_5 = {}^2V_5$<br>$^1V_1 = {}^1V_1$ | $= 2n+1$ | 1 | ... | ... | ... | $2n+1$ | | | | | | | | | | | | |
| 4 | ÷ | $^2V_5 \div {}^2V_4$ | $^1V_{11}$ | $^2V_5 = {}^0V_5$<br>$^2V_4 = {}^0V_4$ | $= \frac{2n-1}{2n+1}$ | ... | ... | ... | 0 | 0 | ... | ... | ... | ... | ... | $\frac{2n-1}{2n+1}$ | | | | | | |
| 5 | ÷ | $^1V_{11} \div {}^1V_2$ | $^2V_{11}$ | $^1V_{11} = {}^2V_{11}$<br>$^1V_2 = {}^1V_2$ | $= \frac{1}{2} \cdot \frac{2n-1}{2n+1}$ | ... | 2 | ... | ... | ... | ... | ... | ... | ... | ... | $\frac{1}{2} \cdot \frac{2n-1}{2n+1}$ | | | | | | |
| 6 | − | $^0V_{13} - {}^2V_{11}$ | $^1V_{13}$ | $^2V_{11} = {}^0V_{11}$<br>$^0V_{13} = {}^1V_{13}$ | $= -\frac{1}{2} \cdot \frac{2n-1}{2n+1} = A_0$ | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | 0 | ......... | $-\frac{1}{2} \cdot \frac{2n-1}{2n+1} = A_0$ | | | | |
| 7 | − | $^1V_3 - {}^1V_1$ | $^1V_{10}$ | $^1V_3 = {}^1V_3$<br>$^1V_1 = {}^1V_1$ | $= n - 1 \ (=3)$ | 1 | ... | $n$ | ... | ... | ... | ... | ... | ... | $n-1$ | | | | | | | |
| 8 | + | $^1V_2 + {}^0V_7$ | $^1V_7$ | $^1V_2 = {}^1V_2$<br>$^0V_7 = {}^1V_7$ | $= 2 + 0 = 2$ | ... | 2 | ... | ... | ... | ... | 2 | | | | | | | | | | |
| 9 | ÷ | $^1V_6 \div {}^1V_7$ | $^3V_{11}$ | $^1V_6 = {}^1V_6$<br>$^0V_{11} = {}^3V_{11}$ | $= \frac{2n}{2} = A_1$ | ... | ... | ... | ... | ... | $2n$ | 2 | ... | ... | ... | $\frac{2n}{2} = A_1$ | | | | | | |
| 10 | × | $^1V_{21} \times {}^3V_{11}$ | $^1V_{12}$ | $^1V_{21} = {}^1V_{21}$<br>$^3V_{11} = {}^3V_{11}$ | $= B_1 \cdot \frac{2n}{2} = B_1 A_1$ | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | $\frac{2n}{2} = A_1$ | $B_1 \cdot \frac{2n}{2} = B_1 A_1$ | ......... | $B_1$ | | | |
| 11 | + | $^1V_{12} + {}^1V_{13}$ | $^2V_{13}$ | $^1V_{12} = {}^0V_{12}$<br>$^1V_{13} = {}^2V_{13}$ | $= -\frac{1}{2} \cdot \frac{2n-1}{2n+1} + B_1 \cdot \frac{2n}{2}$ | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ......... | 0 | $\left\{-\frac{1}{2} \cdot \frac{2n-1}{2n+1} + B_1 \cdot \frac{2n}{2}\right\}$ | | | | |
| 12 | − | $^1V_{10} - {}^1V_1$ | $^2V_{10}$ | $^1V_{10} = {}^2V_{10}$<br>$^1V_1 = {}^1V_1$ | $= n - 2 \ (=2)$ | 1 | ... | ... | ... | ... | ... | ... | ... | ... | $n-2$ | | | | | | | |
| 13 | − | $^1V_6 - {}^1V_1$ | $^2V_6$ | $^1V_6 = {}^2V_6$<br>$^1V_1 = {}^1V_1$ | $= 2n - 1$ | 1 | ... | ... | ... | ... | $2n-1$ | | | | | | | | | | | |
| 14 | + | $^1V_1 + {}^1V_7$ | $^2V_7$ | $^1V_1 = {}^1V_1$<br>$^1V_7 = {}^2V_7$ | $= 2 + 1 = 3$ | 1 | ... | ... | ... | ... | ... | 3 | | | | | | | | | | |
| 15 | ÷ | $^2V_6 \div {}^2V_7$ | $^1V_8$ | $^2V_6 = {}^2V_6$<br>$^2V_7 = {}^2V_7$ | $= \frac{2n-1}{3}$ | ... | ... | ... | ... | ... | $2n-1$ | 3 | $\frac{2n-1}{3}$ | | | | | | | | | |
| 16 | × | $^1V_8 \times {}^3V_{11}$ | $^4V_{11}$ | $^1V_8 = {}^0V_8$<br>$^3V_{11} = {}^4V_{11}$ | $= \frac{2n}{2} \cdot \frac{2n-1}{3}$ | ... | ... | ... | ... | ... | ... | ... | 0 | ... | ... | $\frac{2n}{2} \cdot \frac{2n-1}{3}$ | | | | | | |
| 17 | − | $^2V_6 - {}^1V_1$ | $^3V_6$ | $^2V_6 = {}^3V_6$<br>$^1V_1 = {}^1V_1$ | $= 2n - 2$ | 1 | ... | ... | ... | ... | $2n-2$ | | | | | | | | | | | |
| 18 | + | $^1V_1 + {}^2V_7$ | $^3V_7$ | $^2V_7 = {}^3V_7$<br>$^1V_1 = {}^1V_1$ | $= 3 + 1 = 4$ | 1 | ... | ... | ... | ... | ... | 4 | | | | | | | | | | |
| 19 | ÷ | $^3V_6 \div {}^3V_7$ | $^1V_9$ | $^3V_6 = {}^3V_6$<br>$^3V_7 = {}^3V_7$ | $= \frac{2n-2}{4}$ | ... | ... | ... | ... | ... | $2n-2$ | 4 | ... | $\frac{2n-2}{4}$ | ... | $\left\{\frac{2n}{2} \cdot \frac{2n-1}{3} \cdot \frac{2n-2}{3} = A_3\right\}$ | | | | | | |
| 20 | × | $^1V_9 \times {}^4V_{11}$ | $^5V_{11}$ | $^1V_9 = {}^0V_9$<br>$^4V_{11} = {}^5V_{11}$ | $= \frac{2n}{2} \cdot \frac{2n-1}{3} \cdot \frac{2n-2}{4} = A_3$ | ... | ... | ... | ... | ... | ... | ... | ... | 0 | | | | | | | | |
| 21 | × | $^1V_{22} \times {}^5V_{11}$ | $^0V_{12}$ | $^1V_{22} = {}^1V_{22}$<br>$^0V_{12} = {}^2V_{12}$ | $= B_3 \cdot \frac{2n}{2} \cdot \frac{2n-1}{3} \cdot \frac{2n-2}{3} = B_3 A_3$ | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | 0 | $B_3 A_3$ | ......... | ...... | $B_3$ | | |
| 22 | + | $^2V_{12} + {}^2V_{13}$ | $^3V_{13}$ | $^2V_{12} = {}^0V_{12}$<br>$^2V_{13} = {}^3V_{13}$ | $= A_0 + B_1 A_1 + B_3 A_3$ | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ......... | 0 | $\{A_3 + B_1 A_1 + B_3 A_3\}$ | | | | |
| 23 | − | $^2V_{10} - {}^1V_1$ | $^3V_{10}$ | $^2V_{10} = {}^3V_{10}$<br>$^1V_1 = {}^1V_1$ | $= n - 3 \ (=1)$ | 1 | ... | ... | ... | ... | ... | ... | ... | ... | $n-3$ | | | | | | | |
| | | | | | Here follows a repetition of Operations thirteen to twenty-three. | | | | | | | | | | | | | | | | | |
| 24 | + | $^4V_{13} + {}^0V_{24}$ | $^1V_{24}$ | $^4V_{13} = {}^0V_{13}$<br>$^0V_{24} = {}^1V_{24}$ | $= B_7$ | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ......... | ......... | ......... | ...... | ...... | ...... | $B_7$ |
| 25 | + | $^1V_1 + {}^1V_3$ | $^1V_3$ | $^1V_1 = {}^1V_1$<br>$^1V_3 = {}^1V_3$<br>$^5V_6 = {}^0V_6$<br>$^5V_7 = {}^0V_7$ | $= n + 1 = 4 + 1 = 5$<br>by a Variable-card.<br>by a Variable card. | 1 | ... | $n+1$ | ... | ... | 0 | 0 | | | | | | | | | | |

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Fig. 12 Lady Lovelace's program (1843) for calculating the Bernoulli numbers using Babbage's Analytical Engine

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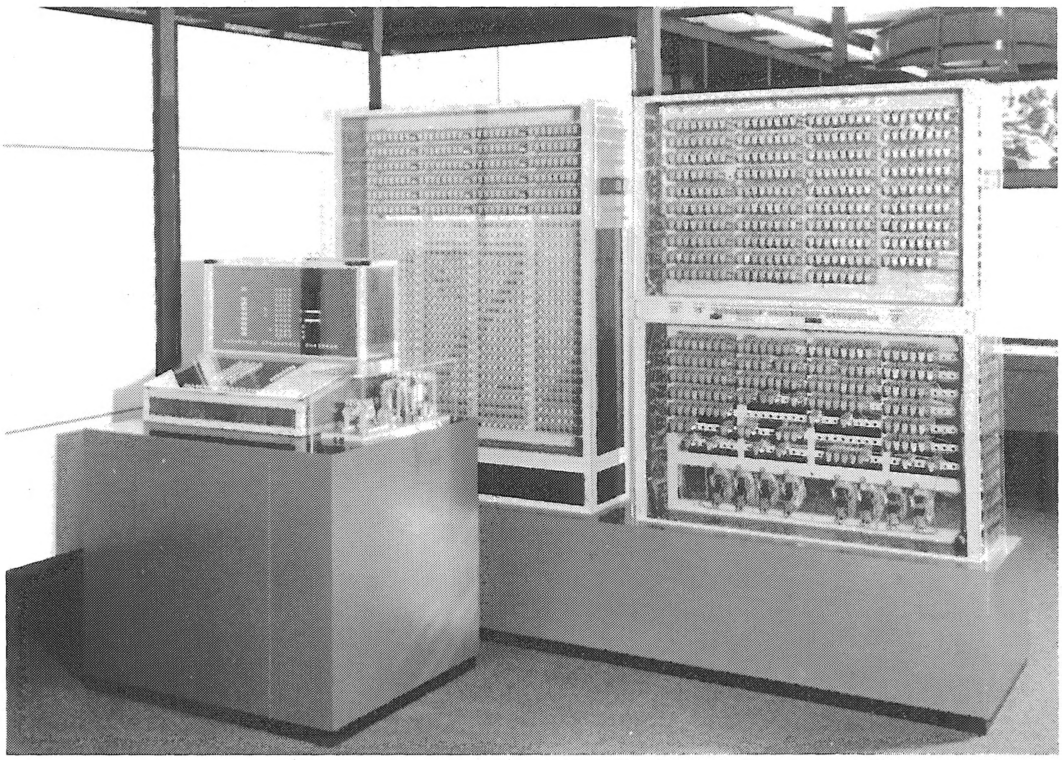
Fig. 13 Punched card machines used in the United States census of 1890



Deutsches Museum

Fig. 14 Zuse's Z3 binary relay computer (1941). A post-war reconstruction.

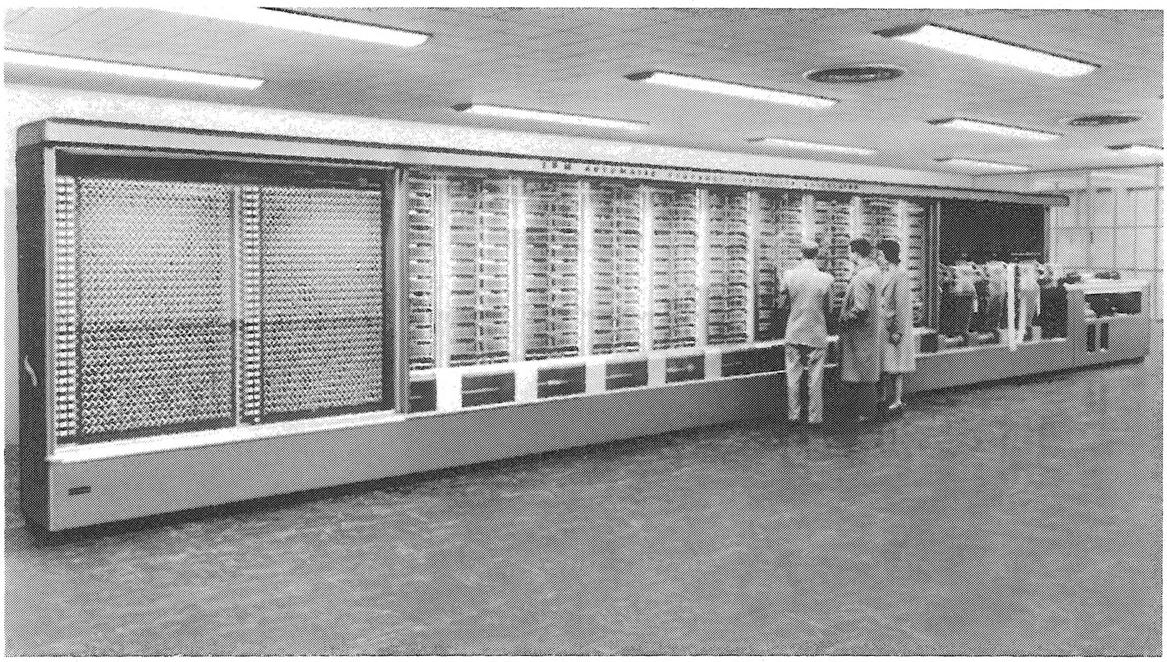


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Fig. 15 The Harvard Mark 1 Calculator (1944)

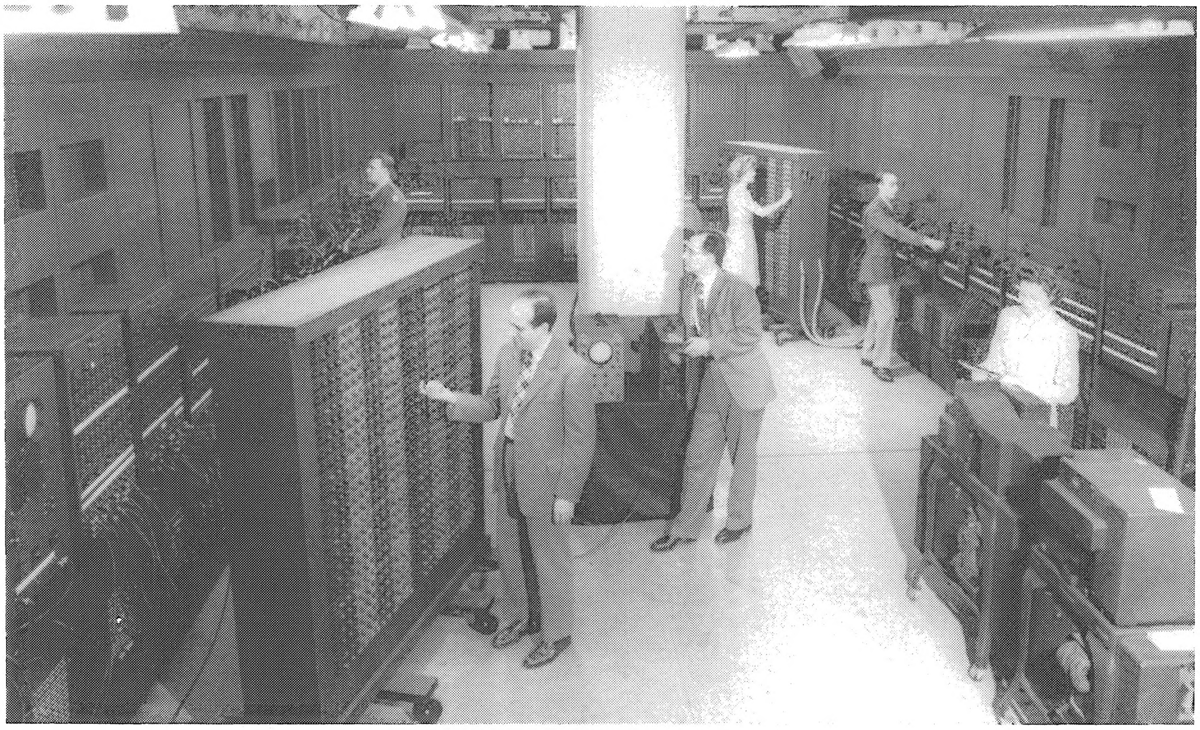

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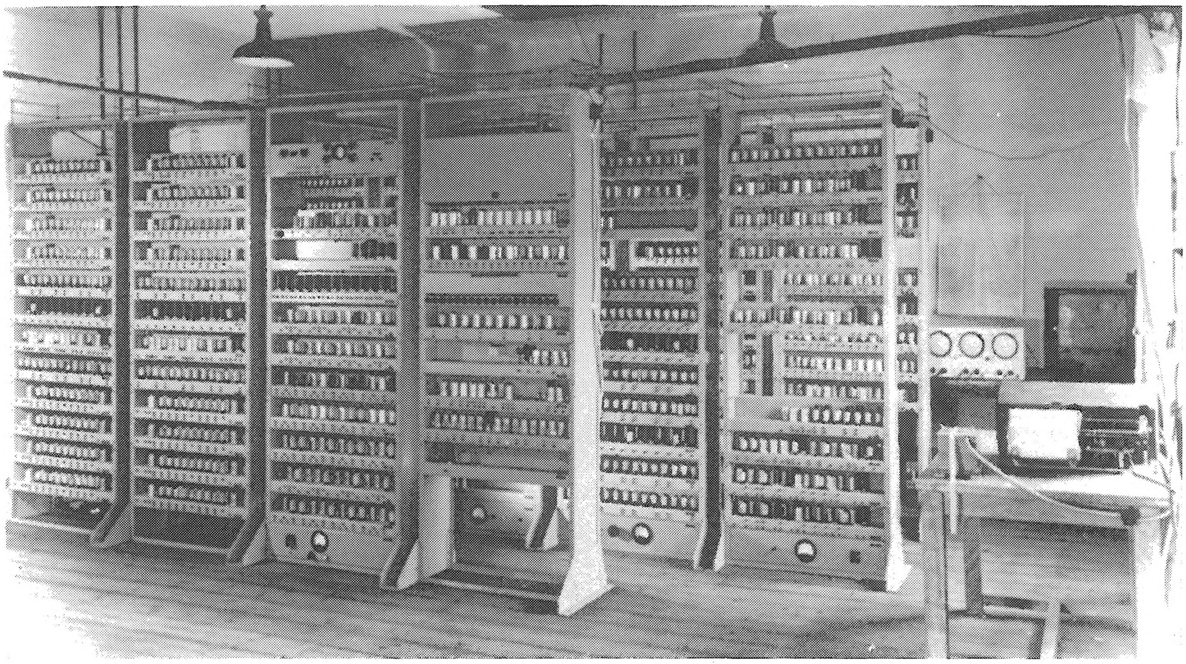
Fig. 16 The ENIAC (1945). J.P. Eckert in left foreground, J.W. Mauchly in centre, H.H. Goldstine in uniform at right.



M.V. Wikes

Fig. 17 The EDSAC (1949)